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SMART TECHNOLOGIES FOR STRUCTURAL HEALTH MONITORING OF AEROSPACE STRUCTURES

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ABSTRACT

This paper presents recent advances in the Structural Health Monitoring (SHM) project within the Consortium for Research and Innovation in Aerospace in Québec (CRIAQ). This project aims at developing technologies and a SHM system for potentially reducing the high costs associated with periodic prescribed inspections of aircraft structures and components. This paper presents an overview of models and strategies developed within this project. Modeling tools describing the dynamic behavior of structures and simulating the effect of defects on modal parameters in low frequency range are first presented. To increase the spatial resolution to defects localization, response in the medium frequency range is investigated and modeling tools are developed and validated for this range. A number of damage detection strategies are proposed and experimentally validated. Strategies based on non-linear response of structures have demonstrated their potential for breathing-like defects detection. Strategies based on differential measurements using piezoelectric elements have demonstrated high sensitivity while requiring dense meshing. Other localized damage detection strategies are also proposed including phased-array antennas and time reversal approaches which are experimentally validated. Piezoelectric sensors and actuators as well as shape memory alloys are explored for passive and active damage monitoring. Early steps in the development of a dedicated micro-accelerometer (MEMS) are also briefly presented.

***Keywords**:* structural health monitoring, vibration modeling and analysis, smart sensors, piezoelectric sensors, shape memory alloys, MEMS sensors, aerospace structures

INTRODUCTION

The Structural Health Monitoring (SHM) project conducted within the Consortium for Research and Innovation in Aerospace in Québec (CRIAQ) aims at reducing the high costs associated with periodic prescribed inspections of aircraft structures through the development of an *in-situ* SHM system [1,2]. SHM is an advanced approach to non-destructive evaluation using integrated technologies. It evaluates in-situ the health status of the structure. The integrated sensing capability would be able to provide structural damage assessment either passively or actively. During normal operation, the envisioned system is further expected to provide real-time *in-situ* structural health monitoring to an on-board data acquisition system. In recent years, the aerospace industry has shown significant interest in the development of a system that would potentially eliminate the need to dismantle aircraft components during periodic inspections. This advanced capability ultimately provides tremendous economic and safety benefits.

This paper presents the main conclusions of Phase I of the project that is oriented towards the development of proof-of-concept prototypes. Models and strategies have been developed for the detection of defects (cracks) in simple structures. Modeling tools that have been developed in the low frequency range to simulate the modal behavior of structures and study the effect of a defect on modal parameters (e.g. frequency, modal shape) are presented in the first section. Modeling tools simulating the behavior of structures in the medium frequency range for increased spatial resolution for defects localization are also presented in this section. Structural health monitoring technologies employing dual function materials, piezoelectric and shape memory alloys, as sensors and actuator are presented in section 2. This is followed by an outline of the preliminary findings on the development of a micro-accelerometer (MEMS) dedicated to SHM. Proposed detection strategies that were tested on experimental test beds (beams and plates) are presented in section 3. Strategies based on non-linear structural response are presented as well as strategies based on differential measurements using piezoelectric elements. Strategies for local damage monitoring using localized sensor arrays are also reported in this section. Phased-array antennas and time reversal approaches are briefly discussed.

MODELING TOOLS

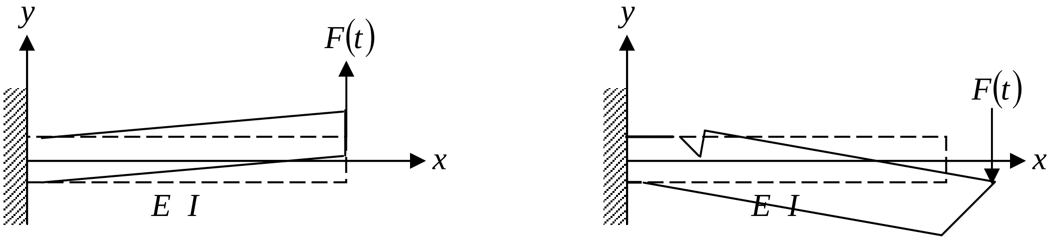
In the open literature, several studies have presented the benefits and limitations of modal-based vibration approaches for structural health monitoring of aircraft structures. Modal-based methods used in low frequency range are limited to the detection of relatively large defects due to the significant stiffness variation. Even if the detection capability is high, the damage localization capability however appears to be low due to the global nature of modal parameters [3]. In high frequency range, using ultrasonic wave propagation approaches, small damages can be detected and localized with the use of a limited number of sensors. The latter approaches are very attractive for structural health monitoring applications, especially using Lamb waves that are known to travel long distances. However, an important challenge for these approaches is the interpretation of complex measurements that highly depend on components material and geometric characteristics [4]. In this work, the medium-frequency range is explored to develop an approach more adapted to visual inspection and to damage sizes in the order of 2,5 cm.

Low-frequency modeling

The presence of cracks in structures influences their intrinsic properties including their dynamic response. This influence is investigated in this project. To validate some of the detection strategies in this frequency range, a beam was modeled incorporating a breathing-crack. The application of a harmonic force (*F(t)*) at the free end of a cantilever beam produces oscillations with compressive and traction stresses (Figure 1). In the case of compression, the crack is closed, and the structure is then considered intact. In traction, the crack opens and the stiffness then decreases as a function of time. The time-varying stiffness $k(t)$ can be described as:

$$k(t) = \begin{cases} k_0 & if \quad t < T/2 \\ k_0\left(1+\delta k \sin(\omega t)\right) & if \quad t > T/2 \end{cases} \qquad (1)$$

where k_0 is the stiffness in close condition and δk is the stiffness variation. This low-frequency model is used to validate the time-frequency approach within this project.



a) b)

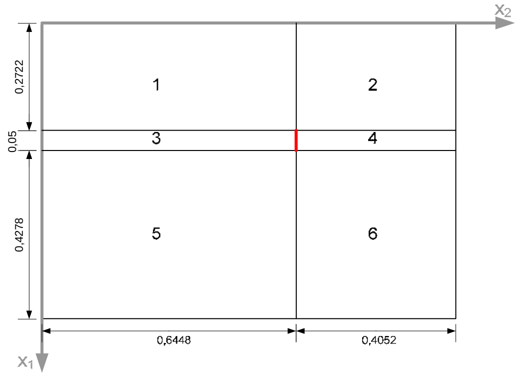
Figure 1: Close (a) and open (b) breathing-crack behavior of the beam over a period *T*.

Medium-frequency modeling

For this frequency range, a modeling tool is developed to describe the vibration behavior of pristine and damaged plates. A Hierarchical Trigonometric Functions Set (HTFS) is used to cover the desired frequency range up to 20 kHz. This HTFS is chosen for its high stability. Using this method, a crack can be described by a discontinuity in the displacement and slope fields between two plate elements [5]. Unlike other models, this one allows analysis of three vibration characteristics that are natural frequencies, modes shapes and flexural wave propagation. The Rayleigh-Ritz approach is used to obtain an approximation of the displacement field. According to orthogonal and dimensionless coordinates ξ and η, the time harmonic solution to the transverse displacement field (*w*) is defined as:

$$w(\xi,\eta,t) = \sum_{m=1}^{M}\sum_{n=1}^{N} q_{mn}(t)\psi_m(\xi)\psi_n(\eta) \qquad (2)$$

where q_{mn} are unknown Rayleigh-Ritz coefficients, $\psi_m(\eta)$ and $\psi_n(\eta)$ are trial functions and *M* and *N* are the number of functions in ξ and η directions [6]. The semi-analytical approach using HTFS provides the capability to use a small number of elements to represent a given desired scenario, Figure 2, without sacrificing accuracy and sensitivity.



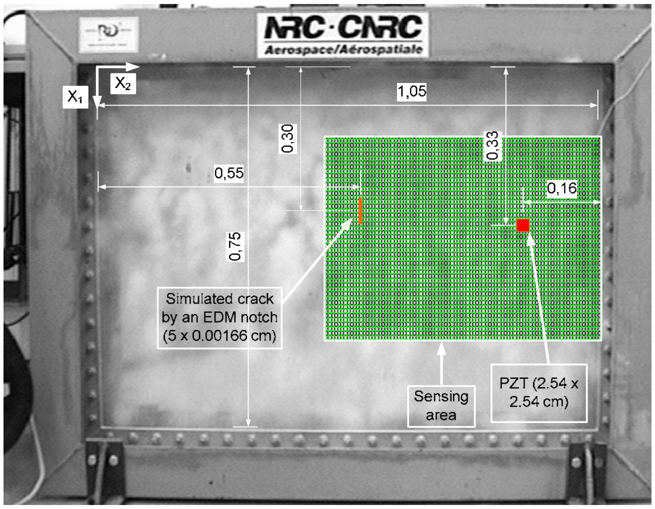
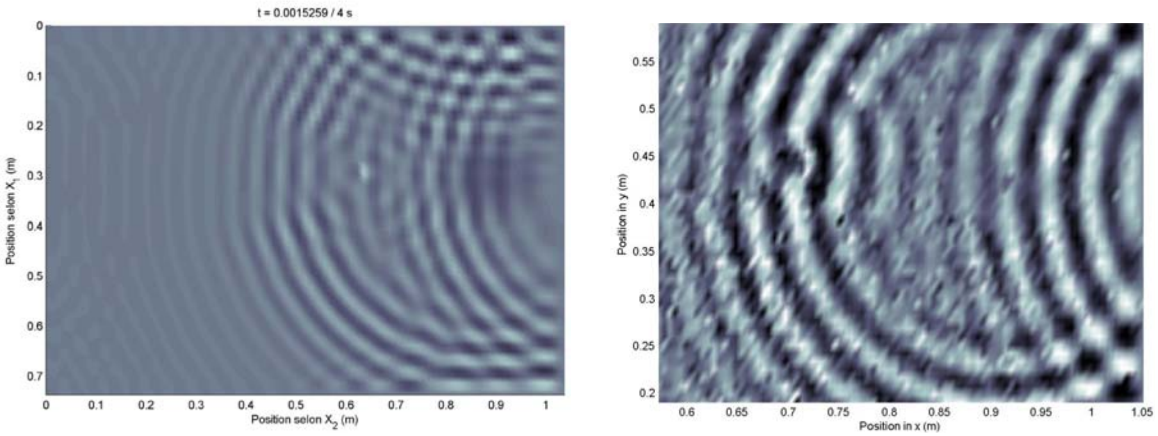


Figure 2: HTFS model construction. **Figure 3**: Experimental cracked plate (in meters).

The frequency response functions of a selected panel with a 5 cm crack subjected to a transverse force applied at the PZT location shown in Figure 3 was computed using the HTFS model. In order to be able to visualize the results, the frequency response functions were computed for 81x61 points in space and 262,144 points in frequency. This frequency domain matrix was transferred in the time domain to produce the impulse responses of the plate. Using a truncated version of this impulse response matrix, the convolution with a force signal (3,5 cycles of a windowed 3950 Hz burst, corresponding to a 5 cm wavelength) can produce the time response of the panel, allowing a propagating wave to be visualized (Figure 4a). Figure 4b presents the corresponding experimental time response of the cracked plate subjected to a bending moment applied by the PZT excitation source driven by the same signal used for numerical result. The wave propagation from the square PZT actuators quickly becomes circular and reacts significantly with the crack where part of the energy is reflected. As anticipated from the numerical results, the crack reacts as a source in the panel when subjected to a wave propagating with a wavelength similar to its characteristic dimension (5 cm).



a) b)

Figure 4: Medium frequency wave propagation in a plate: (a) Simulated (whole plate) and (b) experimental results (meshed region of Figure 3).

SENSORS AND ACTUATORS

Shape memory alloys

As a candidate for use as sensor in this project, shape memory alloy (SMA) is studied. This dual function alloy can be used as sensor and actuator and can be embedded in composite structures during the manufacturing process [7]. This double role has led some researchers to incorporate these SMA wires into composites, with the goal of simultaneously detecting cracks and controlling their propagation [8]. This results in the so-called smart composite materials.

In accordance with project objectives, SMA wires have to be characterized for use as resistive sensors in their martensitic state. The phase transformation temperatures measured for a specific alloy (Ti-50.31at.%Ni) are presented in Figure 5: martensite start (M_s), martensite finish (M_f), austenite start (A_s) and austenite finish (A_f). As shown, when in service at $T < A_s$ (46.5°C), the wires work as resistive sensors in martensitic state; when heated up to $T > A_s$, martensite-austenite transformation occurs, and the wires work as actuators by generating recovery stresses. An 20,3 cm x 30,5 cm unidirectional $[0]_{10}$ Gr/Ep plate was manufactured through a hand lay-up and cured at ambient temperature under vacuum. 20 TiNi wires of 0,5 mm diameter were embedded in at the neutral plane of the plate. The wires were clamped on both ends using a special clamping device in order to avoid shrinkage during the curing process and a hole was used as a stress raiser (Figure 6).

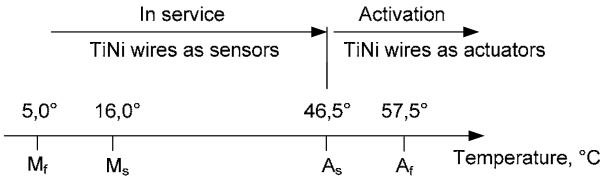


Figure 5: SMA phase transformation.

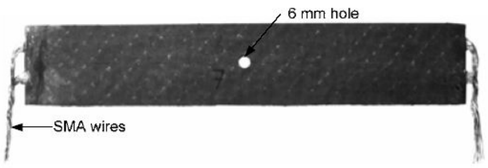


Figure 6: SMA integration into composite.

Since the use of wires as sensors implies the measurement of electrical resistance variation while the sensor is deformed, the gage factor *G* of these wires must be determined. Figure 7 shows experimental curves of stress and electrical resistance as a function of strain applied in order to estimate the gage factor from resistance variation [9].

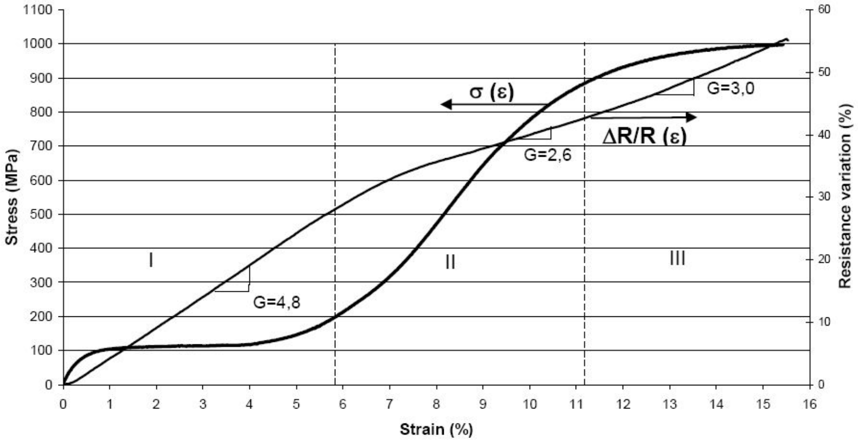


Figure 7: Stress-strain and electrical resistance curves of a 0,5 mm TiNi wire under tensile loading at a 0.02 mm/s rate.

Piezoelectric sensors and actuators

As another dual function materials candidate for sensing in this project is piezoelectric materials. Mathematically, piezoelectricity is described using the following well-known constitutive equations of the piezoelectric material, which define how the stress, strain, charge-displacement, and electric field interact [10]:

$$\{S\} = [s_E]\{T\} + [d]^T\{E\} \qquad (3)$$

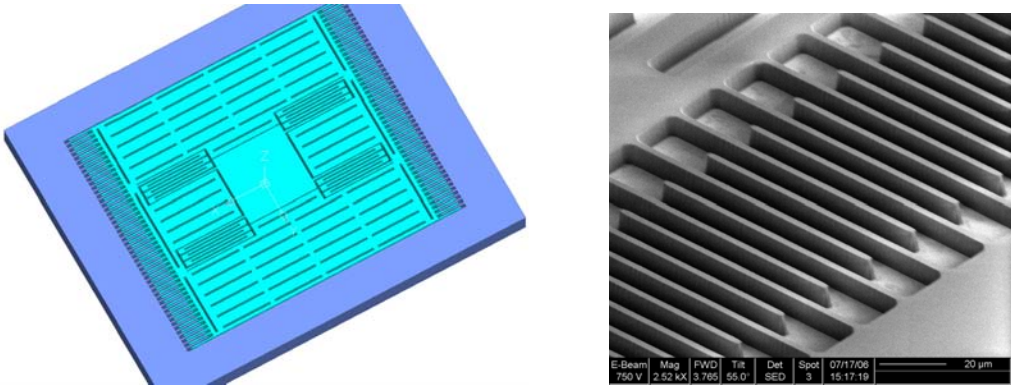
$$\{D\} = [d]\{T\} + [\varepsilon_T]\{E\} \qquad (4)$$

where $\{S\}$, $\{T\}$, $\{E\}$ and $\{D\}$ are the strain, stress, electric field, and electric flux density vectors, respectively. The compliance, piezoelectric coupling and dielectric matrices are denoted by $[s_E]$, $[d]$ and $[\varepsilon_T]$, respectively; where the superscript T denotes the transpose.

In this project, thin piezoelectric ceramics (PZT) or polymers (PVDF) are bonded on the structure. In this work, the piezoelectric constant d_{31} is the key parameter of the voltage-expansion and the useful direction of expansion is normal to that of the electric field.

Micro-accelerometer

To enable strategies for structural health monitoring and damage detection at higher frequencies, a segment of this research project is directed at the measurement of Lamb waves propagation. In order to provide access to out-of-plane information and enable complete measurement of these waves, the design and fabrication of a capacitive type MEMS micro-accelerometer is conducted. Numerous studies have focused on low frequency accelerometers [11] but very few were targeted towards high frequencies. The structure shown in Figure 8(a) consists of comb-drive sensing electrodes with a 5 μm gap between the fingers. The micro-accelerometers were designed such that the mechanical resonant frequencies ranged from 3 kHz to 5 kHz. In order to generate high sensitivity, high-aspect ratio comb-drive electrodes with 100 μm thick fingers were realized on a silicon-on-insulator (SOI) wafer using the deep reactive ion etching (DRIE) technology. Figure 8b) shows a 7 μm thick developed photoresist having the comb structure.



a) b)

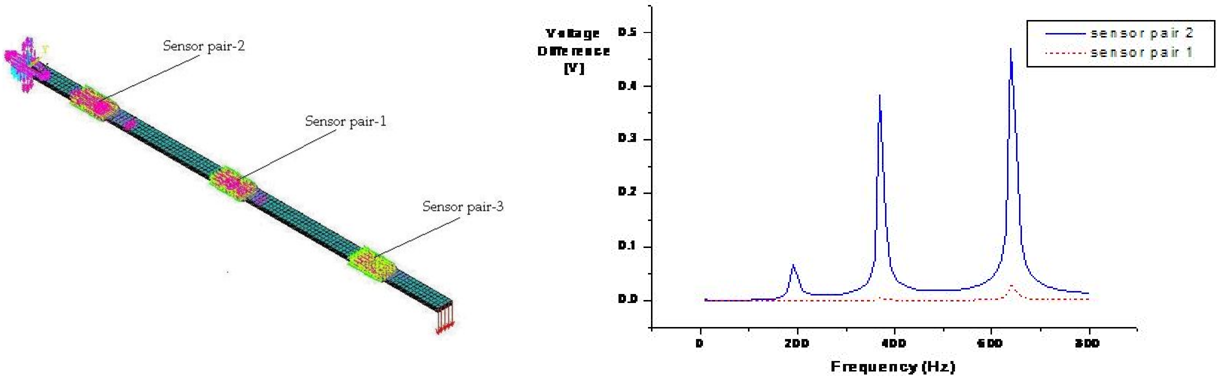
Figure 8: Capacitive micro-accelerometer. (a) Design with comb-drive sensing electrodes and serpentine springs. (b) SEM picture of photoresist for DRIE mask.

DETECTION STRATEGIES

Modal approach

Consider a cantilever beam structure, shown in Figure 9a, on which 4 cm long polymer piezoelectric collocated strips are bonded on both sides at the same axial location. The strips are polarized in the thickness direction. The contact between the piezoelectric strips and the structure surface is assumed to be ideal. In the presence of a crack, experienced strains on the two sides of the structure will be different; hence induced voltage difference is generated in the piezoelectric strips. By measuring the voltage difference between the two piezoelectric strips, the presence of cracks can be predicted [12].

Modal analysis results for a 600 mm x 20 mm x 5 mm beam with a 1,25 mm deep crack located at 130 mm from the fixed end are shown in Figure 9b for a frequency sweep between 0 and 800 Hz. When the sensor pair is close to a crack, the induced voltage difference has peaks that are large enough to be detected. These peaks appear at frequencies that are close to the plate's natural frequencies, and that correspond to the higher bending modal shapes. While being very sensitive to the presence of the crack, this method however requires a dense meshing of the surface of the structure.

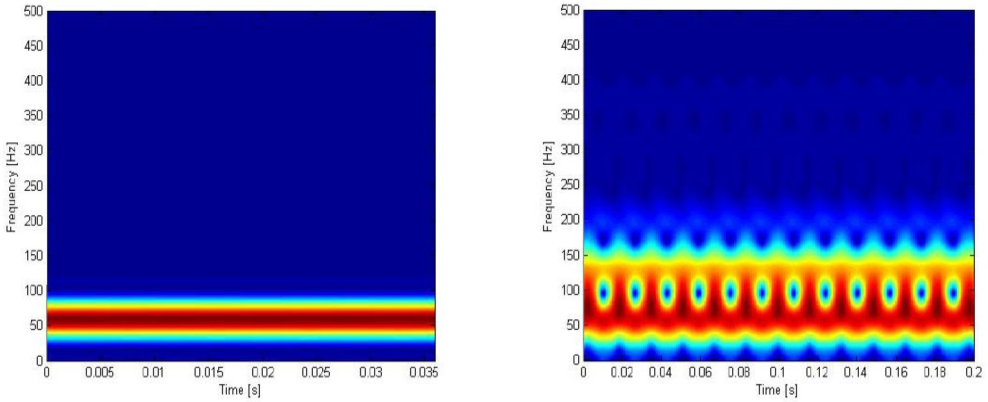


a) b)

Figure 9: Sensing configuration (a) and voltage difference at the sensor pairs with a crack located at 130mm from the fixed end (b).

Time-frequency approach

This proposed strategy consists of exciting the structure with a harmonic excitation at half of the natural frequency of the cracked beam [13] of Figure 1. The undamaged beam shows a pure harmonic vibration while the cracked beam exhibits a more complex periodic vibration in the time domain. The Fourier transform of time signals shows that in the frequency domain, the undamaged beam only shows a response at the excitation frequency while the cracked beam exhibit the excitation frequency and its harmonics due to the non-linear behavior of the stiffness. Furthermore, the second harmonic of the excitation frequency fits with the natural frequency and consequently is amplified. An analysis in the time-frequency domain (Short Time Fourier Transform) is shown in Figure 10. The spectrogram is effective in order to give information on the amplitude modulation at the natural frequency along the time when a crack is present while the signal is purely harmonic at the excitation frequency for a healthy beam.



a) b)

Figure 10: Spectrogram of healthy (a) and cracked (b) beams for an excitation at half of natural frequency.

Wave propagation approaches

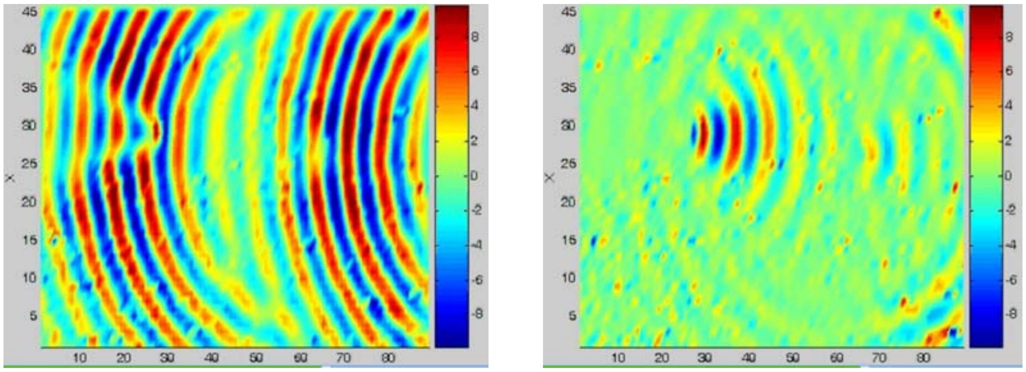
Wave decomposition: Structural intensity for flexural waves (power flow) can be expressed in terms on normal velocity using plate properties. Under free field assumption, the flexural wave field (v) can be seen as a superposition of independent plane waves propagating in different directions, a_m:

$$v(x,y) = \sum_m V_m \exp[-jk(x\cos a_m + y\sin a_m)] \qquad (5)$$

where $k = (\omega)^{1/2}(M_s / B)^{1/4}$ is the structural wavenumber in the plate, ω is the angular frequency, M_s is the mass per unit area of the plate and B is the unit flexural stiffness of the plate, with orthogonal conditions on V_m. Several techniques exist to measure the active structural intensity in directions m with arrays of sensors or laser vibrometry [14]. Recent work has shown the interest of using wave decomposition strategies to detect delamination in composite plates by analysing the phase of the waves [15].

In order to demonstrate the applicability of this approach, the normal velocity of the plate from Figure 3 was measured and processed. The scan area was 0,42 m x 0,58 m and the number of points was 47 x 89. The sampling frequency was 51,2 kHz.

Figures 11a and 11b present the leftward propagating and rightward propagating components of the flexural waves in the plate, as obtained from measurements on the plate. A diffraction effect by the damage can clearly be seen in Figure 11a and an echo effect can also be seen in Figure 11b.



a) b)

Figure 11: Wave propagating components. Leftward (a) and rightward (b).

Beamforming: The simplest array configuration to estimate the Direction of Arrival (DOA) of a propagating wave is the rectangular uniform array shown in Figure 12. The distance between the piezoelectric sensors is denoted d_1 and d_2 in reference to the X_1 and X_2 axis respectively.

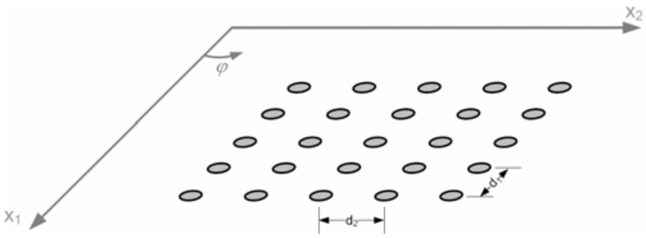


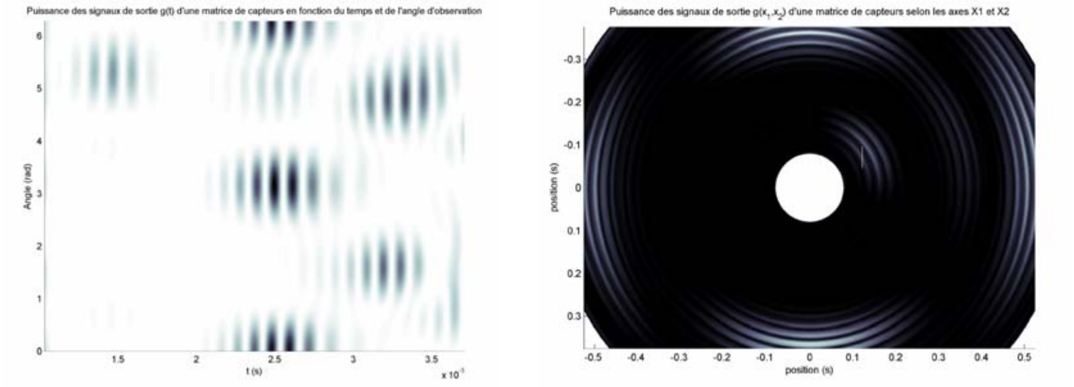
Figure 12: Piezoceramic 5x5 array sensors for the beamforming approach.

In order to demonstrate the application of this approach, the propagation of plane, non-dispersive, waves has been studied. The signal provided by each sensor can be expressed by $f_{m_1m_2}(t)$. As presented in [16], the signals obtained from the sensors are weighted and added so that the *g(t)* function is obtained:

$$g(t) = \frac{1}{M_1 M_2}\sum_{m_1=1}^{M_1}\sum_{m_2=1}^{M_2} f_{m_1m_2}(t+\tau_{m_1m_2}) \tag{6}$$

where $\tau_{m_1m_2}(t)$ is a delay introduced to steer the sensitivity pattern of the array in a given direction $\varphi : \tau_{m_1m_2}(t) = \frac{x_{1_{m_1}}}{c}\cos(\varphi) + \frac{x_{2_{m_2}}}{c}\sin(\varphi)$.

The potential of the approach to localize a crack in a plate has been evaluated by laser vibrometer velocity measurements on a 5x5 uniform rectangular array centered on the plate of the experimental setup presented in Figure 3 [17]. A pair of piezoceramic actuators of 1,25 cm diameter is located at the center of the plate to generate the excitation signal. Figure 13a presents the squared amplitude of *g(t)* for the 5x5 array as a function of time and angle. Based on the knowledge of the wave speed of the excitation signal, Figure 13b presents the polar representation of the same function, reconstructed from a time-space coordinate transformation. The effect of the crack can clearly be seen in Figure 13b.



a) b)

Figure 13: Cartesian (a) and polar (b) representations of the experimental 5x5 array signal.

Time reversal: Time Reversal methods are based on the fact that it is possible to reverse wave propagation in time. In the early 90's significant number of experiments was conducted and reported the ability to detect scatters in high and low dispersive medium [18]. In terms of structural vibrations, improvement of solvers during the past years allows to consider Time Reversal method for both structure vibrations and acoustic propagation at high frequency [19]. In this work, the DORT method is applied to a finite plate to localize a single crack at low frequency [20].

The plate presented in Figure 3 is used to demonstrate the application of time reversal in this project. A Doppler laser vibrometer is used to measure the velocity at each point considered (center of the ceramic). The sources points are configured as an array centered on the plate (see Figure 14). Each piezoceramic sensor is a 13 mm in diameter circular ceramic with 0,5 mm in thickness. The distance between two actuators is 15 mm.

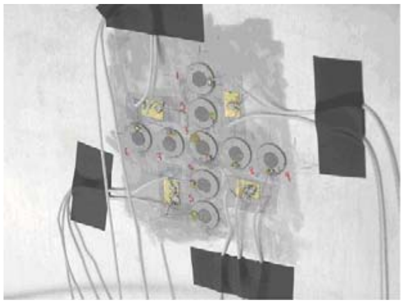
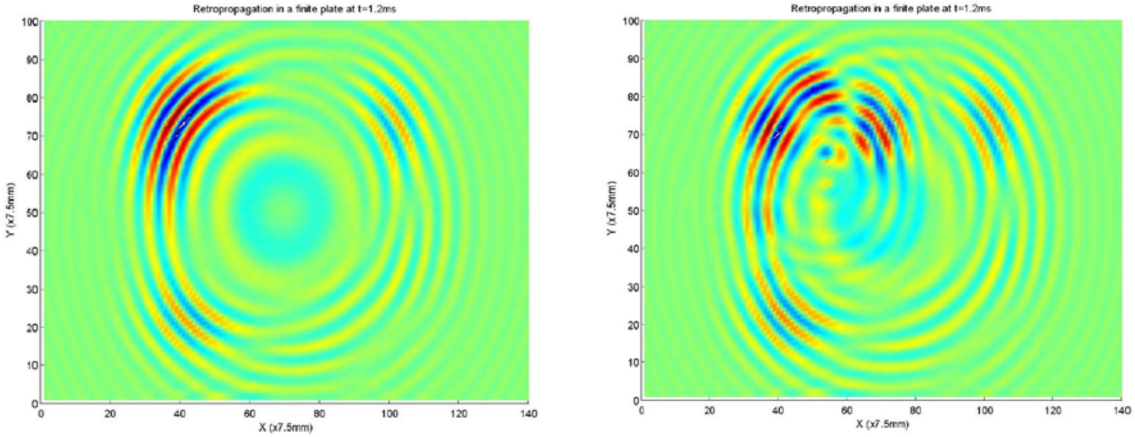


Figure 14: Piezoceramic array for the time reversal approach.

The main array eigenvector was used to retropropagate into pristine and damaged plates. The retropropagation was done using a finite difference model implemented in Simulink. This model was chosen for convenience to calculate the wave propagation in time without having to compute the time responses from frequency response functions given by the model using HTFS. It is then possible to monitor what is retropropagating in the structure. Two retropropagations were done, one using a simple homogeneous undamaged clamped plate and the other using a damaged plate. In the case of the damaged plate, a crack was introduced in the finite difference scheme at the same location as the experimental plate. Figure 15 shows the retropropagation in time of the eigenvector found using experimental results. Figure 15a is in a pristine plate, and Figure 15b is in a damaged plate. These results also confirm that it is possible to make the crack radiate by concentrating emitting energy from the array to the location where the scatter has been found.



a) b)

Figure 15: Retropropagation in a pristine (a) and damaged (b) plate from experimental measurements.

CONCLUSIONS

A number of modeling tools in the low and medium frequency ranges have been developed and validated for damage detection in plates. These models allowed us to test the developed detection strategies and optimize sensing and actuation configurations. The medium frequency model has been presented to simulate the vibration behavior of a cracked thin plate. Numerical investigation of the natural frequencies and mode shapes showed the relatively low effects of the damage on the plate vibration response. Using the same model, the propagation of flexural waves has also been investigated and clearly shows the scattering due to the crack in the plate. A number of sensing and actuation technologies have been proposed for application to SHM within Phase I of this project. Piezoelectric sensors and actuators, as well as shape memory alloys have been used on test benches. A micro-accelerometer has also been specified and designed for application to SHM at higher frequencies. Detection strategies have been presented that can be applied to an integrated SHM sensing and actuation configuration. Strategies based on non-linear response of the structures have demonstrated their potential for breathing-like defects detection. Strategies based on differential measurements using piezoelectric elements have demonstrated high sensitivity while requiring dense meshing of the surface. Other strategies have been proposed to use localized sensors arrays to monitor specific areas of the structures. Beamforming antennas and time reversal approaches have been experimentally validated. The time reversal approach demonstrated that it is possible to obtain some information about the location of the crack using information extracted from the main eigenvector of the time reversal operator.

Future work in Phase II of this project will focus on the implementation of a SHM system on a specific test structure. Models will be exploited to simulate a wide range of cracks, sensor and excitation types, sizes and locations including multiple site damages. This will optimize different configurations of sensor arrays and signal processing strategies to assess the severity of damages and to evaluate their locations.

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