Semicond. Sci. Technol. **19** (2004) L103–L105 PII: S0268-1242(04)83019-0

LETTER TO THE EDITOR

Analytical model for a current density versus voltage relation in n+ υ n+ devices

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Received 5 July 2004, in final form 4 August 2004
Published 15 September 2004
Online at stacks.iop.org/SST/19/L103
doi:10.1088/0268-1242/19/10/L02

Abstract
A new approach for the analytical model of n+ υ n+ devices is developed to express the effects of space–charge-limited current and high field carrier mobility saturation. The analytical solution is compared with a complete numerical resolution and successfully applied to experimental results. We show that laser-diffused resistors, fully compatible with modern CMOS processes, are well described by our analytical model up to high current density levels.

1. Introduction

The current density versus voltage *J–V* relation of n+ υ n+ or p+ π p+ devices has been extensively studied in the past [1–6]. Most models use the drift current approach with injected and intrinsic charge carriers but no simple analytical *J–V* relation has been introduced. The goal of this work is to develop a simple analytical current density versus voltage relationship for 1D trap-less n+ υ n+ or p+ π p+ silicon devices. Previous studies only give sets of relations under various asymptotic conditions [1, 6]. With our approach, we have successfully obtained a direct current density versus voltage relation that can be used to describe the electrical behaviour of the laser-diffused resistors that we recently introduced. The effects of carriers' mobility saturation and space–charge-limited currents are included in the model.

The theoretical model is summarized in section 2 and the analytical solution with its approximations is then discussed. The hypothesis made in regard with the dimensions of our laser-diffused resistors and the voltages used are validated by a complete numerical solution. The model is then successfully applied to experimental results and the physical meaning of the calculated model's parameters is discussed in section 3.

2. Theoretical model

2.1. Model's ground

For simplicity, let us consider a 1D n+ υ n+ structure. Single carrier transport can be described by the drift-diffusion equation where the diffusion current is ignored by a field-dependent mobility expression based on the Canali *et al* model [11]. An impurities' concentration-dependent low-field mobility model was also used [12]. Mobility saturation is implicitly assumed since important electric fields are applied on small integrated circuit geometries, as for laser-diffused resistors. Electric fields of the order of 10^4 V cm^{-1} are easily encountered in the conduction regimes of CMOS microelectronics devices of sub-micron dimensions. The charge-trap density is considered small enough to be ignored and current continuity is implied ($\mathrm{d}j/\mathrm{d}x = 0$):

$$J = nq\frac{\mu_0 E}{1 + |E/E_C|}, \tag{1}$$

where E_C expresses the electric field necessary to observe mobility degradation and μ_0 the low-field mobility. At low applied electric field E, we observe an ideal Ohmic relation. By combining this equation with Poisson's equation, where all

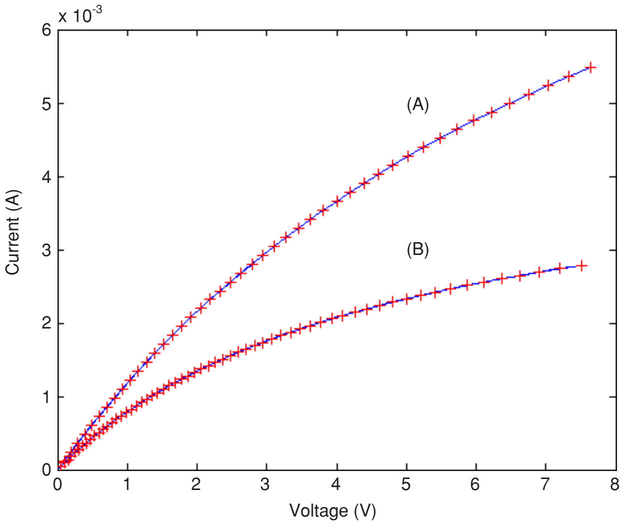

Figure 1. Current–voltage (I–V) plots of the analytical approximation (line) and numerical solution (crosses) of equation (3) for a typical laser-diffused resistor: (A) $N_D = 5 \times 10^{18}$ cm^{-3}, $L = 0.6$ μm and (B) $N_D = 1 \times 10^{18}$ cm^{-3}, $L = 1.7$ μm. (This figure is in colour only in the electronic version)

the impurities N_D are ionized (at room temperature), we obtain the differential equation

$$J = q\left(N_D - \frac{\varepsilon}{q}\frac{\mathrm{d}}{\mathrm{d}x}E\right)\frac{\mu_0 E}{1 + |E/E_C|}. \tag{2}$$

We then make the 'virtual cathode approximation' [1] that stipulates that the electric field is equal to 0 at $x = 0$ and a transcendent solution is obtained by integrating the differential equation (2):

$$-\hat{j}\hat{x}(1/\hat{j} - 1)^2 = (1/\hat{j} - 1)\hat{e} + \ln(1 - (1/\hat{j} - 1)\hat{e}), \tag{3}$$

where we have used the following dimensionless variables:

$$\hat{j} \equiv \frac{J}{qN_D\mu_0 E_C}, \tag{4a}$$

$$\hat{e} \equiv E/E_C, \tag{4b}$$

$$\hat{x} \equiv \frac{-xN_D q}{\varepsilon \cdot E_C}. \tag{4c}$$

In the past studies, the solution presented in (3) was investigated only in certain asymptotic conditions because of its nature and of the complexity of its numerical resolution. Unfortunately, most of the current–voltage (I–V) plots of a typical laser-diffused resistor are within the transition between pure Ohmic conduction and the effect of mobility degradation and space–charge-limited current. This transition cannot be only evaluated in asymptotic conditions.

2.2. Numerical evaluation of the solution

In order to justify the approximations that were made in the development of our analytical model, our solution was compared with a complete numerical resolution of the equation (3). Figure 1 shows the superposition of the two approaches for two extreme cases of laser-diffused resistors' parameters. The goal of this numerical resolution was to find the roots of the transcendent equation (3) by a numerical algorithm and then to evaluate the applied voltage from the space distribution of the electric field.

The numerical solution was performed using a combination of the Steffenson's algorithm and Newton's method [13]. The Newton method was used to determine successively the roots $\hat{e}$ of equation (3) numerically for a large number of given x and $\hat{j}$. As a result we obtained, for each current density $\hat{j}$, the dimensionless electric field spatial distribution. The current–voltage (I–V) plot was then obtained by successive numerical integrations of the smoothly varying electric field over x using the Clenshaw–Curtis quadrature method for as many points in $\hat{j}$ as were needed. Extremely good agreement between the two approaches can be observed which justifies the approximations made and the use of the simplified analytical solution for this model.

While a numerical approach could be used in most result analyses, it is rather cumbersome and lacks flexibility. To obtain the analytical solution, we did the following approximations.

2.3. Mathematical approximation

Since for a positive applied voltage V, the values of J, E and E_C are all negative. The expression

$$-\hat{j}\hat{x}(1/\hat{j})^2 \tag{5}$$

is always negative for all values of $\hat{x}$ and $\hat{y}$. Since the logarithmic term will always dominate the right-hand side of equation (3) for typical values of $\hat{j}$ and $\hat{e}$ we can then approximate our transcendent solution by

$$-\hat{j}\hat{x}(1/\hat{j} - 1)^2 \approx \ln(1 - (1/\hat{j} - 1)\hat{e}). \tag{6}$$

It is now possible to express our dimensionless electric field as a function of the current density and hence find the applied voltage by integration over the device's length L:

$$V = \frac{E_C^2\varepsilon}{N_D q}\frac{-\hat{j}^2}{(\hat{j}-1)^3}\left\{\exp\left(\frac{-\hat{x}_L}{\hat{j}}(\hat{j}-1)^2 - 1\right) + \frac{\hat{x}_L}{\hat{j}}(\hat{j}-1)^2 - 1\right\} \tag{7}$$

where

$$\hat{x}_L \equiv \frac{-LN_D q}{\varepsilon E_C}. \tag{8}$$

We can even further obtain a simplified current density versus voltage relation by neglecting the exponential term (since the argument is often negative for diffused resistor's dimensions):

$$V \approx \frac{E_C^2\varepsilon}{N_D q}\frac{-\hat{j}^2}{(\hat{j}-1)^3}\left\{\frac{\hat{x}_L}{\hat{j}}(\hat{j}-1)^2 - 1\right\} \equiv \frac{E_C^2\varepsilon}{N_D q}\hat{v}. \tag{9}$$

The device resistance can also be calculated from relation (9):

$$R = \left|\frac{\mathrm{d}V}{\mathrm{d}I}\right| = \left|\frac{E_C^2\varepsilon}{N_D q}\frac{1}{Aq\mu_0 N_D E_C}\right| \cdot \left|\frac{\mathrm{d}\hat{v}}{\mathrm{d}\hat{j}}\right|$$
$$\approx \frac{-E_C\varepsilon}{AN_D^2 q^2\mu_0}\left\{\frac{\hat{j}^2(\hat{x}_L - 1) - 2\hat{j}(\hat{x}_L^2 + 1) + \hat{x}_L}{(\hat{j}-1)^4}\right\}. \tag{10}$$

The resistance at zero applied voltage is coherent with Ohm's law:

$$R(\hat{j} = 0) = \frac{L}{qN_D A\mu_0}. \tag{11}$$

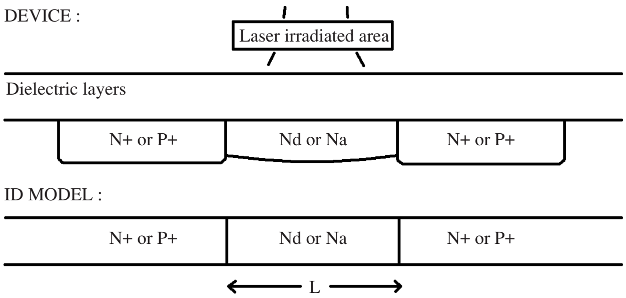

Figure 2. Schematization of the 1D model applied to the geometry of a laser-diffused resistor.

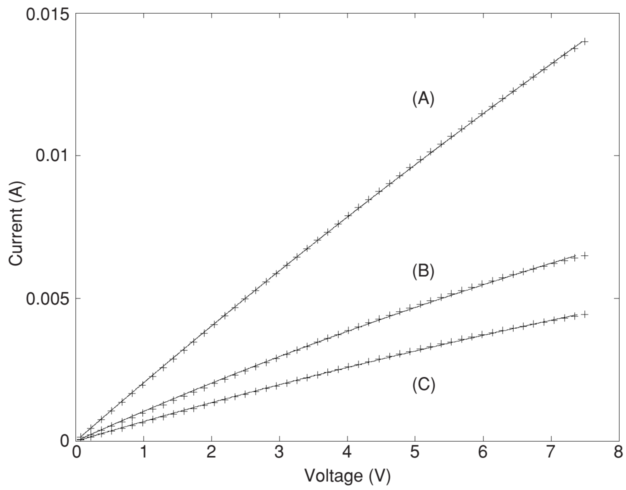

Figure 3. Current–voltage (I–V) plot of the experimental data (crosses) and fitted model (line) for three typical laser-diffused resistors of parameters: (A) $R = 500\ \Omega$, $N_D = 1.45 \times 10^{18}$ cm^{-3}, $L = 0.6 \times 10^{-4}$ cm; (B) $R = 1$ kΩ, $N_D = 5.02 \times 10^{18}$ cm^{-3}, $L = 1.0 \times 10^{-4}$ cm and (C) $R = 1.5$ kΩ, $N_D = 2.66 \times 10^{18}$ cm^{-3}, $L = 0.6 \times 10^{-4}$ cm.

3. Experimental results

The use of laser beams to form connections between highly doped diffusion regions has been introduced and investigated in past studies [7, 8]. Diffused resistors can be made with very precise nominal resistance values by an iterative process [9, 10]. A high energy laser beam is focused between the source and drain of a gateless MOSFET device. The impurities can then diffuse easily in the illuminated melting silicon region, therefore creating a resistor with a very linear current–voltage (I–V) behaviour for typical microelectronics' voltages (below a few volts). The device created can be modelled, in terms of transport behaviour, by an n+ υ n+ or p+ π p+ diode depending on the doping type [2, 6].

Once the impurities have diffused into the gap and the molten silicon has cooled down, the device presents an n+–n–n+ or p+–p–p+ structure with non-abrupt junctions (figure 2). By varying the laser parameters, highly precise I–V curves can be obtained from pulse to pulse. The somewhat simple 1D model presented in section 2 gives good results to evaluate the transport behaviour of the devices created.

Our experimental results were obtained with on-chip laser-diffused resistors of nominal gap between 0.6 μm and 1.4 μm. Four wire electrical measurements where performed on the samples in a Faraday cage at room temperature. The current–voltage (I–V) curves were evaluated up to 7.5 V, the maximum applicable voltage on our test chip before breakdown. The model described in section 2 was applied to fit the experimental curves with only N_D and E_C as free parameters. Figure 3 shows the agreement between the experimental data and the fitted model. The three fitted curves give values of N_D between 1×10^{18} cm^{-3} and 5×10^{18} cm^{-3} which correspond to the expected level of impurities in the melted region (N+ or P+ impurities concentrations in the order of 5×10^{19} cm^{-3}). Values of E_C are slightly higher than the crystalline value, probably because the diffused region is partly poly-crystalline.

4. Conclusion

Experimental data and numerical resolution have demonstrated that a simplified analytical solution of the 1D trap-less n+ υ n+ or p+ π p+ diode model gives good results for modelling the transport behaviour of laser-diffused resistors for modern microelectronics. This demonstration was based on a complete numerical resolution of the initial equation and the application of the analytical solution on experimental data from fabricated on-chip resistors.

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