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Resonant-tunneling lifetime comparison between double-barrier and δ-doped barrier structures

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We examine the resonant-tunneling lifetime of two systems: a double-barrier and a δ-doped barrier structure. Simple analytical expressions for the lifetime, which account for the effective-mass changes between the different regions, are obtained. Results suggest that the δ-doped barrier structure would be faster and would carry a larger peak current density than an identical double-barrier structure.

Interest in resonant-tunneling systems has increased considerably in recent years due to the experimental results of Sollner *et al.*[1] on double-barrier structures. They demonstrated the high-speed response attainable by these devices (frequencies $>10^{12}$ Hz). In order to estimate the frequency limit of such high-speed devices, it is important to know the lifetime of the resonant state.

The structures considered in this study are shown in Fig. 1. Several authors[2-7] have presented analytical expressions for the lifetime of a double-barrier resonant-tunneling (DBRT) structure [Fig. 1(a)]; however, they all assumed equal effective masses for the barrier and the well regions. Others[8-10] have correctly considered the effective-mass difference and obtained the lifetime of the DBRT structure by numerically solving the time-dependent Schrödinger equation. The second structure considered is inspired from Beltram and Capasso,[11] who investigated the effect of deep levels within the barrier region of semiconductor superlattices. The structure shown in Fig. 1(b) consists of a single barrier containing a sheet of impurities at a distance z_0 from the first interface. As shown by Hjalmarson,[12] deep levels could result from the deposition of an atomic layer of highly concentrated shallow impurities. The technique for the deposition of dopants in a single atomic plane (δ doping) was established by Wood *et al.*,[13] and is reviewed by Ploog *et al.*[14]

In this paper we obtain simple analytical expressions for the lifetime of the resonant-tunneling states for both structures while properly accounting for the effective-mass profile of these structures.

We first calculate the global transmission coefficient of the symmetric DBRT structure [Fig. 1(a)]. For simplicity, we assume no externally applied field in the following calculation. The lifetime of the resonant-tunneling state is obtained from the linewidth of the transmission maximum.

Ricco and Azbel[2] have published a discussion of the physics of resonant tunneling, while assuming a single effective mass for the entire structure. However, Bastard[15] has shown that effective-mass jumps modify the boundary conditions which are imposed on the wave functions of the effective-mass Hamiltonian. Therefore we repeat Ricco and Azbel's calculation of the transmission coefficient using the effective-mass Hamiltonian and Bastard's boundary conditions. We obtain, for the symmetric DBRT structure,

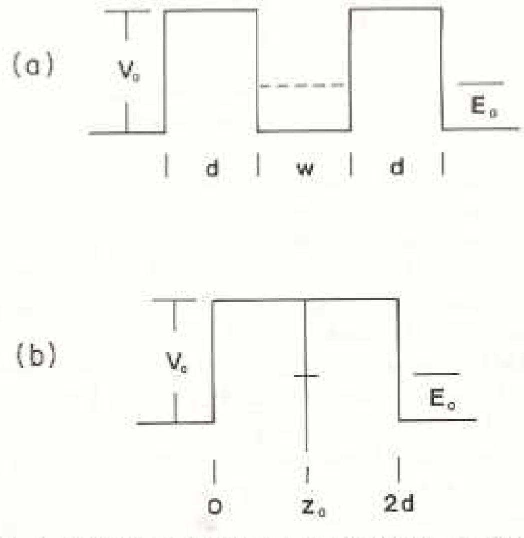

FIG. 1. Conduction-band potential-energy profiles for (a) symmetric double-barrier resonant-tunneling structure and (b) δ-doped barrier structure. In each figure, V_0 is the barrier height, d is the barrier width on either side of the well region, and E_0 is the energy of a bound state for an isolated well ($d\rightarrow\infty$). For (a), w is the well width while, in (b), z_0 is the position of the δ-function potential with respect to the first interface.

$$T=\left[\left[\cosh(2Kd)\cos(kw)+\frac{1}{2}\left[\frac{1}{X}-X\right]\sinh(2Kd)\sin(kw)\right]^2+\left\{\frac{1}{4}\left[X+\frac{1}{X}\right]^2\sin(kw)-\frac{1}{2}\left[\frac{1}{X}-X\right]\left[\sinh(2Kd)\cos(kw)+\frac{1}{2}\left[\frac{1}{X}-X\right]\cosh(2Kd)\sin(kw)\right]\right\}^2\right]^{-1}, \tag{1}$$

where

$$k=[(2m_1^*/\hbar^2)E]^{1/2},\quad K=[(2m_2^*/\hbar^2)(V_0-E)]^{1/2} \tag{2}$$

and

$$X=m_2^*k/m_1^*K\ . \tag{3}$$

Here, m_1^* and m_2^* are, respectively, the effective masses of the well and barrier regions. E is the energy of the incident electron while V_0, d, and w are defined in Fig. 1(a). For $m_1^*=m_2^*=m$, the transmission coefficient obtained is identical to the previously published results.[2,5]

To simplify (1), we assume $Kd \gg 1$ so that the hyperbolic functions may be replaced by $\frac{1}{2}e^{2Kd}$. Substitution in (1) gives

$$T=\left\{\frac{e^{4Kd}}{16}\left[\frac{1}{X}+X\right]^2\left[\cos(kw)+\frac{1}{2}\left[\frac{1}{X}-X\right]\sin(kw)\right]^2-\frac{e^{2Kd}}{8}\left[\frac{1}{X}-X\right]\left[\frac{1}{X}+X\right]^2\sin(kw)\left[\cos(kw)+\frac{1}{2}\left[\frac{1}{X}-X\right]\sin(kw)\right]+\frac{1}{16}\left[\frac{1}{X}+X\right]^2\sin^2(kw)\right\}^{-1}. \tag{4}$$

For off-resonance energies, the first term dominates the transmission coefficient so that $T\propto e^{-4Kd}$. However, the leading term will cancel and the transmission coefficient will approach a maximum when

$$\cos(kw)+\frac{1}{2}\left[\frac{1}{X}-X\right]\sin(kw)=0\ . \tag{5}$$

It is interesting to note that this condition determines the energy levels of an isolated quantum well of width w and of depth V_0.[16]

According to Ricco and Azbel, a symmetric DBRT structure with identical barriers will produce a resonance with a transmission coefficient of unity. However, it is easily verified that, for energies which satisfy (5), $T\sim 1$ but is not unity. Therefore, in order to find the energy of the resonance, E_r, and the half width at half maximum (HWHM) Γ, we will expand the expression of T about an energy E_0 which satisfies (5). Replacing E by $E_0+\Delta E$ where $\Delta E \ll E_0$ and keeping only the leading terms in ΔE, (4) reduces the transmission coefficient to an expression of the form $[A(\Delta E)^2+B\,\Delta E+C]^{-1}$. Setting this expression equal to 1 and $\frac{1}{2}$ will yield quadratic equations whose roots give, respectively, the resonance energy E_r and the HWHM for the DBRT structures, Γ_{DB}. We find

$$\Gamma_{DB}=\frac{8(V_0-E_0)e^{-2K_0d}}{\left[X_0+\dfrac{1}{X_0}\right]\left[\dfrac{k_0w}{2}\dfrac{(V_0-E_0)}{E_0}\left[X_0+\dfrac{1}{X_0}\right]+\dfrac{V_0}{E_0}\right]} \tag{6}$$

and

$$E_r^{DB}=E_0-\frac{\Gamma_{DB}}{2}\left[\frac{1}{X_0}-X_0\right]. \tag{7}$$

Here, k_0, K_0, and X_0 are the values of k, K, and X when $E=E_0$. The resonant-tunneling lifetime τ of a parallel-plane barrier heterostructure of any form is related to the HWHM by[7]

$$\tau=\frac{\hbar}{2\Gamma}\ . \tag{8}$$

The lifetime obtained by combining (6) and (8) is a generalization of the expressions previously published by Payne[4] and Araki[5] who assumed no effective-mass difference between the barrier and well regions.

Let us now consider the δ-doped barrier structure of Fig. 1(b). The formalism is similar to the double-barrier case except one must specify the potential used to describe the impurity layer. Beltram and Capasso[11] have verified that only the symmetry and weight (i.e., the integral) of the potential chosen are important. Therefore we shall use a δ-function potential, as suggested by Beltram and Capasso. To obtain the global transmission coefficient, we follow the same procedure detailed above while using the appropriate boundary conditions at the δ potential.[17] We obtain

$$T=\left\{\left[\cosh(2Kd)-\frac{K_0}{K}\sinh(2Kd)\right]^2+\left[\frac{K_0}{2K}\left[X+\frac{1}{X}\right]\cosh(2Kz_0-2Kd)-\frac{1}{2}\left[\frac{1}{X}-X\right]\left[\sinh(2Kd)-\frac{K_0}{K}\cosh(2Kd)\right]\right]^2\right\}^{-1}, \tag{9}$$

which is given for an arbitrary position z_0 of the δ potential. K_0 is the value of K for $E=E_0$ where E_0 is the energy level of the bound state of an isolated δ potential well. E_0 can be written in terms of the weight of the δ potential and can be considered as a parameter. Our result agrees with the transmission coefficient obtained by Knauer *et al.*[18] who considered metal-insulator-metal (MIM) tunnel junctions and ignored the effective-mass changes between the various regions. In order to be able to compare the results with the DBRT structure previously treated, we consider a symmetric structure where the δ potential lies in the middle of a barrier of thickness $2d$ (i.e., $z_0=d$). We can solve (9) by using the same procedure detailed for the DBRT structure to obtain

$$\Gamma_\delta=\frac{8(V_0-E_0)e^{-2K_0d}}{(X_0+1/X_0)} \tag{10}$$

and

$$E_r^\delta=E_0-\frac{\Gamma_\delta}{2}\left[\frac{1}{X_0}-X_0\right], \tag{11}$$

where k_0, K_0, and X_0 are evaluated for $E=E_0$. The resonant-tunneling lifetime for the δ-doped barrier is also expressed as in (8) where Γ is given by (10).

As an example, let us consider a $GaAs/Ga_{0.7}Al_{0.3}As$ structure which gives $V_0=0.33$ eV, $m_2^*=0.092m_0$, and $m_1^*=0.067m_0$.[19] In order to compare both structures, we chose $w=43.5$ Å which yielded $E_0=0.10$ eV for the DBRT structure, and used this same value of E_0 for the δ-doped barrier structure. Figure 2 shows the HWHM and the resonant-tunneling lifetime as a function of barrier width d. The symbols shown are values of the HWHM obtained numerically from the expressions of the total transmission coefficients [see (1) and (9)]. The agreement between the analytical expressions (lines) and the numerical results (symbols) is excellent when $d>25$ Å. However, for thin barriers ($d<25$ Å), the approximation $Kd \gg 1$ used to calculate Γ becomes less valid.

From Fig. 2, one can also notice that τ_δ is almost one order of magnitude smaller than τ_{DB}. This is easily verified by taking the ratio of the two lifetimes which gives

$$\frac{\tau_{DB}}{\tau_\delta}=\frac{k_0w}{2}\frac{V_0-E_0}{E_0}\left[\frac{1}{X_0}-X_0\right]+\frac{V_0}{E_0}. \tag{12}$$

This quantity is strictly greater than one since E_0 must be less than V_0. Therefore δ-doped barrier structures are faster than equivalent DBRT structures.

Furthermore, (12) also implies that Γ_δ is greater than Γ_{DB}. Also, the current density through a resonant-tunneling system can be written as[6]

$$J=q\int\left[\frac{2E}{m_1^*}\right]^{1/2}T(E)n(E)dE, \tag{13}$$

where $n(E)$ is the net distribution of incoming particles. Therefore the broader peak in the transmission coefficient of the δ-doped barrier would lead to a larger current density than the DBRT structure. A more complete analysis of the current density of these structures will be published later.

We have obtained analytical expressions for the energy position, the HWHM, and consequently the resonant-tunneling lifetime for DBRT and δ-doped barrier structures. These analytical expressions agree very well with the numerical results. A comparison between the two structures suggests that a δ-doped barrier would be faster and would permit a larger peak current density than an identical double-barrier structure.

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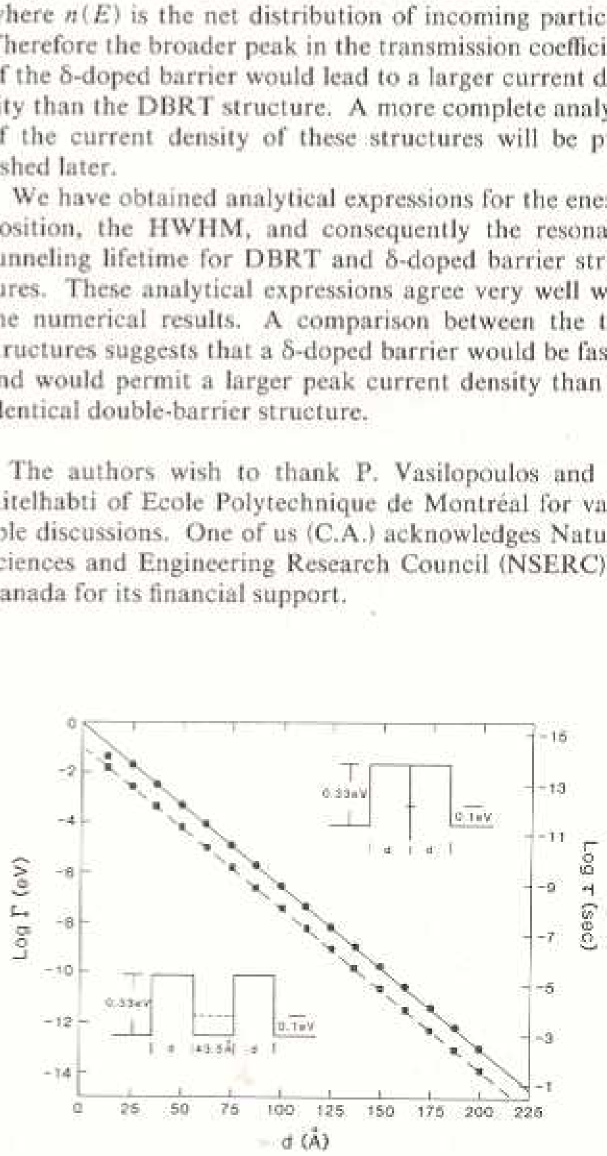

FIG. 2. Logarithm of the half-width at half maximum (HWHM) of the transmission peak, Γ, and the resonant-tunneling lifetime τ as a function of barrier width d. The solid (δ-doped structure) and dashed (DBRT structure) lines are the approximate values given by (10) and (6), respectively. The circles and squares are the numerical results obtained from the total transmission coefficient [i.e., (9) and (1)].

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